EXERCISE – IV HINTS & SOLUTIONS

Sol.1 Let a Point $(\lambda - 2, 2\lambda + 3, 3\lambda + k)$

In y – z plane $x = 0 \Rightarrow \lambda = 2$

A (0, 7, 6 + k)

In x – y lane $z = 0 \Rightarrow \lambda = -k/3$

$$B\left(\frac{-k}{3} - 2, \frac{-2k}{3} + 3, 0\right)$$

$$\overline{A}.\overline{B} = 0 \Rightarrow \left(\frac{-2k}{3} + 3\right) = 0$$

$$k = \frac{9}{2}$$

Sol.2 $\overrightarrow{PQ} = (-1, -2, -1)$

$\overrightarrow{PR} = (1, -5, -1)$

$\overrightarrow{PS} = (5, -2, 2)$

$$\text{volume} = \frac{1}{6}\,[\overrightarrow{PQ}\quad \overrightarrow{PR}\quad \overrightarrow{PS}] = \frac{1}{2}$$

Sol.3 **(i)**

$(x - 1)^2 + (y - 3)^2 + (z + 6)^2 + (x - 2)^2 + (y - 4)^2 + (z - 2)^2 = 72$

$\Rightarrow x^2 + y^2 + z^2 - 3x - 7y + 4z - 1 = 0$

$$\text{Center}\left(\frac{3}{2}, \frac{7}{2}, -2\right)$$

(ii) $$r = \sqrt{\left(\frac{3}{2} - 1\right)^2 + \left(\frac{7}{2} - 3\right)^2 + (2 - 6)^2}$$

$$= \sqrt{\frac{33}{2}}$$

(iii) plane: $2x + 2y - z + 3 = 0$

$$d = \left|\frac{2(3/2) + 2(7/2) + 2 + 3}{3}\right| = 5$$

Sol.5 M (1, 2, 0)

$\overrightarrow{OP} = (1, 2, 3)$

$\overrightarrow{OM} = (1, 2, 0)$

$$\cos\theta = \frac{3}{\sqrt{14}} \Rightarrow \theta = \cos^{-1}\frac{3}{\sqrt{14}}$$

$$\cos\phi = \frac{1}{\sqrt{5}} \Rightarrow \phi = \cos^{-1}\frac{1}{\sqrt{5}}$$

Sol.6 $$\frac{x}{1} = \frac{y}{1} = \frac{z-1}{-2}$$

$\vec{a}$ = DR'S = (1, 1, –2) Fixed point P(0, 0, 1)

$x + y + z = 1$; $\vec{n} = (1, 1, 1)$

$\vec{n} \cdot \vec{a} = 1 + 1 - 2 = 0$

& point P satisfy the plane

$\Rightarrow$ line lies is the plane.

Let the line $\frac{x}{a} = \frac{y}{b} = \frac{z-1}{c}$

$$\cos\theta = \frac{1}{\sqrt{6}} = \left|\frac{a + b - 2c}{\sqrt{a^2 + b^2 + c^2}\,\sqrt{6}}\right|$$

squareing

$3c^2 + 2ab = 4c(a + b)$ ....(1)

let the point is plane (1, 0, 0)

$\Rightarrow$ condition of copalanarity

$$\begin{vmatrix} 1 & 0 & -1 \\ a & b & c \\ 1 & 1 & -2 \end{vmatrix} = 0 \quad ....(2)$$

Solve (1) & (2) and get (a, b, c)

Sol.7 $$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z-3}{c}$$

$$\frac{x+1}{2} = \frac{y-2}{1} = \frac{z+4}{2}$$

Lines are coplaner.

$$\begin{vmatrix} a & b & c \\ 2 & 1 & 2 \\ 2 & 0 & 7 \end{vmatrix} = 0 \Rightarrow 7a - 10b - 2c = 0 \text{ ......(1)}$$

and a + 5b + 4c = 0 ....(2)

from (1) & (2)

$a = k,\ b = k,\ c = -\frac{3}{2}k$

$$\frac{x-1}{k} = \frac{y-1}{k} = \frac{z-3}{-\frac{3}{2}k}$$

$$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-3}{-3}$$

Sol.8 $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

Lines will be coplaner so

$$\begin{vmatrix} a & b & c \\ 2 & 1 & 1 \\ 3 & 3 & 0 \end{vmatrix} = 0 \Rightarrow a = b + c$$

$$\cos 60° = \left| \frac{2a+b+c}{\sqrt{a^2+b^2+c^2}\sqrt{6}} \right|$$

$\Rightarrow 2b^2 + 2c^2 + 5bc = 0$

$\Rightarrow (b + 2c)(2b + c) = 0$

$b = -2c$ or $b = -c/2$

$a = -c$ $a = c/2$

$$\frac{x}{-c} = \frac{y}{-2c} = \frac{z}{c} \quad \text{or} \quad \frac{x}{c/2} = \frac{y}{-c/2} = \frac{z}{c}$$

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{-1} \quad \text{or} \quad \frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$$

Sol.9 $\frac{x+2}{3} = \frac{y+3}{2} = \frac{z+4/3}{5/3} = \lambda$

$$Q\left(3\lambda - 2,\ 2\lambda - 3,\ \frac{5\lambda-4}{3}\right)$$

$$\overrightarrow{PQ} = \left(3\lambda,\ 2\lambda - \frac{9}{2},\ \frac{5\lambda+8}{3}\right)$$

$\vec{n} = (4, 12, -3)$

$\overrightarrow{PQ} \cdot \vec{n} = 0 \Rightarrow \lambda = 2$

$$\overrightarrow{PQ} = \left(6, -\frac{1}{2}, 6\right)$$

distance = $|\overrightarrow{PQ}| = \frac{17}{2}$

Sol.10 Direction of line

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -3 \\ 3 & -4 & 2 \end{vmatrix} = -6\hat{i} - 13\hat{j} - 17\hat{k}$$

line : $\frac{x-1}{-6} = \frac{y+2}{-13} = \frac{z+3}{-17}$

or $\frac{x-1}{6} = \frac{y+2}{13} = \frac{z+3}{17}$

Sol.11 $\frac{x-4}{a} = \frac{y+14}{b} = \frac{z-4}{c}$

direction of intersecting line = $\vec{n}_1 \times \vec{n}_2 = (-6, 5, -8)$

Put z = 0 in both the planes

3x + 2y = 5

x – 2y = –1 x = 1, y = 1

P (1, 1, 0)

Another line $\frac{x-1}{-6} = \frac{y-1}{5} = \frac{z-0}{-8}$

–6a + 5b – 8c = 0

both lines will be coplanar

$$\Rightarrow \begin{vmatrix} a & b & c \\ -6 & 5 & -8 \\ 3 & -15 & 4 \end{vmatrix} = 0 \qquad \Rightarrow 4a = 3c$$

If $a = k,\ c = \frac{4}{3}k,\ b = \frac{10}{3}k$

$$\frac{x-4}{3} = \frac{y+14}{10} = \frac{z-4}{4}$$

Sol.12 **(a)**

$\overrightarrow{PQ} = (0, 1, 2)$ $\overrightarrow{PR} = (1, 1, 4)$

$\overrightarrow{PQ} \times \overrightarrow{PR} = 2\hat{i} + 2\hat{j} - \hat{k}$

Area = $\frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \frac{3}{2}$

(b)

$2(x - 1) + 2(y - 0) - (z + 1) = 0$

$2x + 2y - z - 3 = 0$

$\frac{2}{3}x + \frac{2}{3}y - \frac{1}{3}z = 1$

(c)

$x = 0, z = 0$

$y = 3/2$

point $\left(0, \frac{3}{2}, 0\right)$

(d)

dir of line will be along the normal of plane

$$\frac{x-2}{2} = \frac{y-1}{2} = \frac{z-3}{-1}$$

Sol.13 Direction of intersection line

$= \vec{n}_1 \times \vec{n}_2$

put z = 0 in both planes

$x - 2y = 1$ $\quad x = 3, y = 1$

$x + 2y = 5$

point (3, 1, 0)

line : $\frac{x-3}{2} = \frac{y-1}{3} = \frac{z-0}{4}$

variable point $(2\lambda + 3, 3\lambda + 1, 4\lambda)$

$2(2\lambda + 3) + 2(3\lambda + 1) + 4\lambda + 6 = 0$

$\Rightarrow \lambda = -1$

point (1, –2, –4)

Sol.14 A(2, 0, 0) ; B(0, 3, 0) ; C(0, 0, –5)

normal of plane = $\overrightarrow{AB} \times \overrightarrow{AC}$

$= (-15, -10, 6)$

Equation of plane

$-15(x - 2) - 10(y - 0) + 6(z - 0) = 0$

$\frac{x}{2} + \frac{y}{3} + \frac{z}{(-5)} = 1$

Area = $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$= \frac{19}{2}$

Sol.15 direction of line = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{vmatrix}$

$= -2\hat{i} + 4\hat{j} - 2\hat{k}$

equation of line

$$\frac{x-2}{-2} = \frac{y+1}{4} = \frac{z-3}{-2}$$

$$\frac{z-2}{1} = \frac{y+1}{-2} = \frac{z-3}{1}$$

Sol.16 Normal of plane = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 5 \\ 1 & -1 & 1 \end{vmatrix}$

$= 2\hat{i} + 3\hat{j} + \hat{k}$

equation of plane

$2(x - 1) + 3(y - 2) + 1(z - 0) = 0$

$2x + 3y + z + 4 = 0$

Sol.17 coplanar $\Rightarrow \begin{vmatrix} -3 & 2 & 1 \\ 1 & -3 & 2 \\ 1 & p-7 & 5 \end{vmatrix} = 0 \Rightarrow p = 1$

$$\frac{x-1}{-3} = \frac{y-1}{2} = \frac{z+2}{1} = \lambda$$

& $\dfrac{x}{1} = \dfrac{y-7}{-3} = \dfrac{z+7}{2} = \mu$

$-3\lambda + 1 = \mu$ ....(1)

$2\lambda + 1 = -3\mu + 7$ ....(2)

$\lambda - 2 = 2\mu - 7$ ....(3)

$\lambda = -3/7$ & $\mu = 16/7$

Point of intersection $\left(\dfrac{16}{7}, \dfrac{1}{7}, \dfrac{-17}{7}\right)$

Normal of plane = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} \begin{matrix} = (7, 7, 7) \\ = (1, 1, 1) \end{matrix}$

Equation of plane $(x-1) + (y-1) + (z+2) = 0$

$x + y + z = 0$

Sol.18 $\vec{n}_1 = (1, -2, 3)$; $\vec{n}_2 = (2, 3, -4)$

Directionof line = $\vec{n}_1 \times (\vec{n}_1 \times \vec{n}_2)$

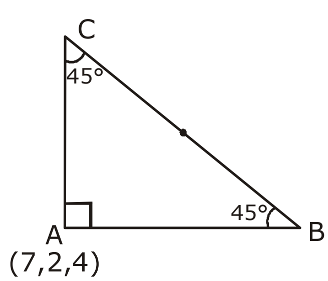
$= (-44, -10, 8)$

$\dfrac{x-7}{-44} = \dfrac{y-2}{-10} = \dfrac{z+1}{8}$

or $\dfrac{x-7}{22} = \dfrac{y-2}{5} = \dfrac{z+1}{-4}$

Sol.19 Let the DR's of AB = (a, b, c)

$\cos 45° = \left|\dfrac{5a + 3b + 8c}{\sqrt{a^2 + b^2 + c^2}\,\sqrt{96}}\right|$



$48(a^2 + b^2 + c^2) = (5a + 3b + 8c)^2$ .....(1)

condition of coplanarity

$\begin{vmatrix} 7+6 & 2+10 & 4+14 \\ a & b & c \\ 5 & 3 & 8 \end{vmatrix} = 0$

$\begin{vmatrix} 13 & 12 & 16 \\ a & b & c \\ 5 & 3 & 8 \end{vmatrix} = 0$ ....(2)

Solve (1) & (2) & get a, b, c

Sol.20 Line & plane are $\perp^r$ to each other

image of (1, 2, –3) in the plane is foot of $\perp^r$

(α, β, γ)

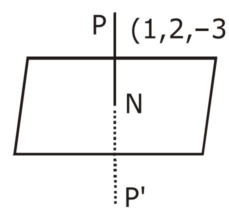
$\dfrac{\alpha-1}{3} = \dfrac{\beta-2}{-3} = \dfrac{\gamma+3}{10} = \lambda$

$N(3\lambda + 1, -3\lambda + 2, 10\lambda - 3)$

$\Rightarrow 3(3\lambda + 1) - 3(-3\lambda + 2) + 10(10\lambda - 3) = 26$

$\Rightarrow \lambda = 1/2$

$N\left(\dfrac{5}{2}, \dfrac{1}{2}, 2\right)$



$\dfrac{P + P'}{2} = N \Rightarrow P' = 2N - P$

$\Rightarrow P' \equiv (4, -1, 7)$

equation of line

$\dfrac{x-4}{9} = \dfrac{y+1}{-1} = \dfrac{z-7}{-3}$

Sol.21 Normal of plane = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ 2 & 5 & 4 \end{vmatrix} = 2\hat{i} - 4\hat{j} + 4\hat{k}$

plare will passes through (1, 0, 0)

$\Rightarrow 1(x-1) - 2y + 2z = 0$

$x - 2y + 2z = 1$